

University of Saskatchewan
Department of Mathematics & Statistics
Mathematics 110.3

Time: 3 hours

Final Examination

9am, December 9, 1999

CLOSED BOOK EXAMINATION - NO CALCULATORS ALLOWED

Name: _____ Student #: _____ Math 110 section #: _____

PART I

Questions in this part will be marked right or wrong. Please carefully write your answers in the spaces provided.

[7]

1. (a) $\lim_{x \rightarrow 2} x - 4 =$ _____

(b) $\lim_{t \rightarrow -2} \frac{t + 2}{2t^3 + 7t^2 + 7t + 2} =$ _____

(c) $\lim_{s \rightarrow 0} \sqrt{2} =$ _____

(d) $\lim_{h \rightarrow 0} \frac{(2 + h)^5 - 32}{h} =$ _____

(e) $\lim_{\theta \rightarrow 0} \frac{\sin(2\theta)}{\sin(\theta)} =$ _____

(f) $\lim_{x \rightarrow 1^+} \frac{x - 1}{|x - 1|} =$ _____

(g) $\lim_{x \rightarrow 4^-} \frac{x^2 - 2}{x^2 - 5x + 4} =$ _____

[3]

2. (a) At what x -value(s) does the graph of $y = \frac{x - 1}{x^2 - 4x + 3}$ have a vertical asymptote? _____

(b) Find $\lim_{x \rightarrow \infty} \frac{3x^2 - 7x + 22}{1 - 3x^2} =$ _____

(c) Find $\lim_{x \rightarrow -\infty} \frac{2x + 12}{\sqrt{4x^2 + 2x + 10}} =$ _____

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[20] 3. Carry out the indicated differentiations. It is not necessary to simplify your answers.

(a) If $p(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{4}x^4$, then $p'(x) =$ _____

(b) If $y = \tan(3x)$, then $\frac{dy}{dx} =$ _____

(c) If $w = 3^{(t-1)}$, then $\frac{dw}{dt} =$ _____

(d) If $f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$, then $f'(t) =$ _____

(e) If $g(x) = \frac{1}{1+x^3}$, then $g'(x) =$ _____

(f) If $u = \frac{t^2 - 1}{t^2 + t + 1}$, then $\frac{du}{dt} =$ _____

(g) If $f(t) = t^2(\ln t)(\sin t)$, then $f'(t) =$ _____

(h) If $y = \ln \left[\frac{x^2 - 4}{x^2 + 1} \right]$, then $\frac{dy}{dx} =$ _____

(i) If $h(s) = s^{\cos s}$, then $h'(s) =$ _____

(j) If $V = \frac{4}{3}\pi r^3$, then $\frac{dV}{dr} =$ _____

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- [10] 4. (a) What is the domain of the function $\ln(1 - x^2)$? _____
- (b) Find an antiderivative $F(x)$ of $f(x) = x^2 - \sqrt{x}$ that satisfies $F(1) = 1$. _____
- (c) Complete the following definition. A function f is called decreasing on an interval I if _____ whenever $x_1 < x_2$ in I .
- (d) Complete the following statement: If $f'(x) = g'(x)$ for all x in an interval (a, b) , then there is a constant C so that _____ for $x \in (a, b)$.
- (e) Complete the following statement: If f has a local maximum or minimum at c , and if $f'(c)$ exists, then _____.

PART II

Please provide carefully written answers to questions 5 through 14 in an answer booklet.

- [6] 5. Use the formal definition of the derivative (that is; work from first principles) to find the slope of the tangent line to the graph of $y = 2 + x^2$ at the point $(2, 6)$. (No marks will be given for using the rules of differentiation.)
- [6] 6. An oil pipeline under a large lake starts to leak. The oil comes to the surface and forms a growing circular shaped slick with a uniform thickness of 2 cm. At a given time the slick is observed to have a radius of 100 meters and the radius is increasing at a rate of 2 meters per minute. At what rate is the oil leaking from the pipe?
- [6] 7. Consider $f(x) = xe^{-x}$.
- (a) What is the domain of $f(x)$? At what point(s) x is $f(x) = 0$?
- (b) Identify the intervals where $f(x)$ is increasing or decreasing.
- (c) Identify the intervals where $f(x)$ is concave up or down.
- (d) Identify any local maxima or minima of $f(x)$.
- (e) Sketch a graph of $y = xe^{-x}$.
- (f) Based on your graph, what do you think $\lim_{x \rightarrow \infty} xe^{-x}$ might be?

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- [6] 8. A cylindrical can without a top is required to hold $8\pi \text{ cm}^3$ of liquid. What is the smallest possible area of material that can be used in making this can? (Assume there is no wastage in constructing the can.)
- [6] 9. Find the equation of the tangent line to the graph of $x^{2/3} + y^{2/3} = 5$ at the point $(-8, 1)$.
- [6] 10. (a) Let $f(\theta) = \sin(2\theta)$. Find $f'(\theta)$, $f''(\theta)$, $f^{(3)}(\theta)$ and $f^{(4)}(\theta)$.
(b) What is $f^{(9)}(0)$?
- [6] 11. Use one step of Newton's method to estimate the cube root of 30. That is, let $f(x) = x^3 - 30$ and estimate the root of $f(x)$ by starting with an initial guess of $x_1 = 3$ and applying one step of Newton's method. Leave your answer in fractional form.
- [6] 12. Let $f(x) = \frac{x+2}{1+x^2}$ for $x \in [-2, 2]$.
(a) Find all points $x \in [-2, 2]$ that are critical numbers for f .
(b) What are the absolute maximum value and absolute minimum value of $f(x)$ for $x \in [-2, 2]$.
- [6] 13. (a) Show that the equation $x - 1 + \sin\left(\frac{\pi}{2}x\right) = 0$ has at least one solution in the interval $[0, 1]$.
(b) How many solutions to the equation in (a) are there? Give justifications for your answers. (Do not try to calculate the value of any solutions.)
- [6] 14. Let c be a constant and $f(x) = x^3 + 3cx^2 + 3x + 2$. Find those values of c for which $f(x)$ has no local maximum. Verify your claim.

The End